

# Advanced Math

## 9a-5

### Binomial Theorem

#### Combination Function:

$${}_nC_r = \frac{n!}{(n-r)!r!} \quad n \text{ and } r \text{ are integers such that } n \geq 0, r \geq 0, n \geq r$$

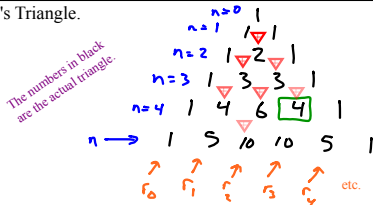
The way we read this function is  $n$  Choose  $r$ .

Evaluate. (pg 755).

$$1) {}_5C_3 = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = 10$$

This one is read Five Choose Three. The output (10) tells how many combinations of choice are possible. There are ten ways to choose three of five things.

#### Pascal's Triangle.



See the pattern? Start with the top three ones. Then add the top two numbers to continue the pattern. Each row starts with one.

Evaluate using Pascal's Triangle:  ${}_5C_3 = 4$

Each node in Pascal's Triangle is the output to a Combination function.

#### Binomial Theorem :

$$(x + y)^n = \sum_{r=0}^n {}_nC_r x^{n-r} y^r$$

Use the binomial theorem to expand and simplify the expression.

$$21) (y - 2)^4 = \sum_{r=0}^4 {}_4C_r y^{4-r} (-2)^r = {}_4C_0 y^4 (-2)^0 + {}_4C_1 y^3 (-2)^1 + {}_4C_2 y^2 (-2)^2 + {}_4C_3 y^1 (-2)^3 + {}_4C_4 y^0 (-2)^4$$

$$= 1y^4(1) + 4y^3(-2) + 6y^2(2) + 4y(-2)^3 + 1y^0(-2)^4$$

$$= y^4 - 8y^3 + 24y^2 - 32y + 16$$

Using Pascal's Triangle is the fastest way to FOIL binomials of large powers.

Notice, each circled coefficient is a number from Pascal's Triangle. In fact, 1 4 6 4 1 is the entire row for  $n = 4$ .

#### Expand and simplify.

27)  $(x - y)^5 =$

$$1x^5y^0 - 5x^4y^1 + 10x^3y^2 - 10x^2y^3 + 5xy^4 - 1y^5$$

The negative will alternate because each is  $(-y)^n$  and powers will be  $n = 0, 1, 2, 3, \dots$ . Thus even terms are +, odd terms are -.

Assignment:  
pg. 755  
2-10 even,  
16-48 every 4th.