Advanced Math

9a-5
Binomial Theorem

<u>Combination Function</u>:

$${}_nC_r = \frac{n!}{(n-r)!\,r!} \qquad n \text{ and } r \text{ are integers such that } n \geq 0,\ r \geq 0,\ n \geq r$$

The way we read this function is *n* Choose *r*.

Evaluate. (pg 755).

1) $_5C_3 = \dfrac{5!}{2!\,3!} = \dfrac{5 \cdot 4 \cdot \cancel{3!}}{2 \cdot 1 \cdot \cancel{3!}} = 10$

This one is read Five Choose Three. The output (10) tells how many combinations of choice are possible. There are ten ways to choose three of five *things*.

Pascal's Triangle.

The numbers in black are the actual triangle.

$n=0$: 1

$n=1$: 1 1

$n=2$: 1 2 1

$n=3$: 1 3 3 1

$n=4$: 1 4 6 $\boxed{4}$ 1

$n \longrightarrow$ 1 5 10 10 5 1

r_0 r_1 r_2 r_3 r_4 etc.

See the pattern? Start with the top three ones. Then add the top two numbers to continue the pattern. Each row starts with one.

Evaluate using Pascal's Triangle: $\boxed{_4C_3} = 4$

Each node in Pascal's Triange is the output to a Combination function.

<u>Binomial Theorem</u> :

$$(x+y)^n = \sum_{r=0}^{n} {}_nC_r\, x^{n-r} y^r$$

Use the binomial theorem to expand and simplify the expression.

21) $(y-2)^4 =$

$$\overbrace{{}_4C_0\, y^{4-0}(-2)^0}^{r=0} + \overbrace{{}_4C_1\, y^{4-1}(-2)^1}^{r=1} + \overbrace{{}_4C_2\, y^{4-2}(-2)^2}^{r=2} + \overbrace{{}_4C_3\, y^{4-3}(-2)^3}^{r=3} + \overbrace{{}_4C_4\, y^{4-4}(-2)^4}^{r=4}$$

$$1y^4(1) + \boxed{4}y^3(-2)^1 + \boxed{6}y^2(-2)^2 + \boxed{4}y^1(-2)^3 + \boxed{1}y^0(-2)^4$$

$$= y^4 - 8y^3 + 24y^2 - 32y + 16$$

Using Pascal's Triangle is the fastest way to FOIL binomials of large powers.

Notice, each circled coefficient is a number from Pascal's Triangle. In fact, 1 4 6 4 1 is the entire row for $n = 4$.

Expand and simplify.

27) $(x-y)^5 =$

$$1x^5\cancel{y^0} - 5x^4y^1 + 10x^3y^2 - 10x^2y^3 + 5xy^4 - 1y^5$$

The negative will alternate because each is $(-y)^n$ and powers will be $n = 0, 1, 2, 3\ldots$ Thus even terms are +, odd terms are -.

Assignment:
pg. 755
2-10 even,
16-48 every 4th.